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CONTENTS

These are slides, notes and other materials for AST560 Spring 2021 course for the portion of the class taught by Ammar Hakim. These lectures focus on finite-volume and discontinuous Galerkin schemes for partial differential equations (PDEs), specifically fluid mechanics (Euler equations) and plasma physics (MHD equations, multi-fluid equations and the Vlasov-Maxwell system). We will look at schemes suited to shock dominated flows rather than problems on resistive time-scales. Vast majority of laboratory, space and astrophysical problems have complex interactions of shocks, turbulence and magnetic fields and the schemes in these lectures will help you solve equation to study such phenomena.

The final couple of lectures will be an introduction to implicit methods to solve and couple diffusive and other non-ideal physics to hyperbolic PDE solvers.

As reading for the lectures please read Chapters 1, 2, 3 and 5 in [LeVeque1992]. Some notes and references to material not completely covered in class:

- van Dyke's "Album of Fluid Motion" is an excellent source of beautiful pictures of fluid flow. See this link for a PDF of an older version of the book.
- See eigensystem of Euler equations listed here and Maxwell equations here.
- For ideal MHD equations the eigensystem is very complex. A listing is in [RyuJones1995] but it may be a good idea to rederive this and cross-check.

In addition to the complexity of the physics and the mathematics of numerical methods, computational physics software is very complex. In addition to the physics itself being complicated (multiple spatial and temporal scales, highly nonlinear physics, coupling between models, huge number of unknowns, etc), the algorithms required may be sophisticated, difficult to implement efficiently and may require complex data-structures. In general, to achieve expertise in this topic one needs to write a lot of code under the supervision of a "Master Craftsman". It is hard to learn the real details of the field any other way.

A nice summary of the concepts of the finite-volume schemes is given in the paper "A one-sided view" by Roe, LeVeque and van Leer.

Contents

CHAPTER

ONE

LECTURE NOTES AND SLIDES

Notes covering aspects of the finite-volume schemes we covered in class. Work in progress!

- Lecture 1 slides
- Lecture 2 slides
- Lecture 3 slides
- Lecture 4 slides
- Lecture 5 slides
- Lecture 6 slides
- Lecture 7 slides

CHAPTER TWO

HOMEWORK

My goal in setting homework is to review material covered in class. Due to the pandemic forced remote learning it is hard to focus and learn as much as one really should in a class like this. I hope these homework sets allow a review of the material and improve learning in these very trying and difficult situations.

- Homework Set 1

Fig. 1: “Gacha Life” character drawn by Sophia Hakim (10 YO) to represent homework character Fivo L. Hacker (“Finite Volume Hacker”).

CHAPTER THREE

THE BIG PICTURE

To understand why computational methods have become central in plasma physics we need to look at a few big-picture issues.

In particular:

- What are the cutting-edge research questions in computational plasma physics?
- What is the relationship between modern numerical methods and experiments and observations? (That is, why care about this stuff in the first place? Can simulations *predict* rather than *postdict*?)
- How to incorporate "real-world" effects into simulations? (For example, boundary conditions, atomic physics, etc)

One can look at computational physics in two ways: as an end in itself, and as a tool for applications. Both of these are important!

As an end in itself:

- The first sits between applied mathematics and theoretical physics. The goal is to design efficient numerical methods to solve equations from theoretical physics.
- The goal here is the numerical method itself: what are its properties? Does it faithfully represent the underlying physics? Does it run efficiently on modern computers? Research into modern numerical methods (including structure preserving methods) fall into this category.
- Usually, besides the fun of solving complex equations (and writing code), the goal is to gain deeper understanding of underlying physics. **Some theoretical questions can only be answered with computer simulations.**
- This is a perfectly legitimate research area even if no connection to experiments is made, but only satisfies the curiosity of the researchers and helps one gain a better understanding of the physics.

As a tool for applications:

- The second is to look at the computational physics as providing tools to understand/design experiments or observations.
- Note that a large number of routine calculations are needed to build modern experiments (heat-transfer, structural analysis, basic fluid mechanics, equilibrium and stability calculations, etc). **Such routine calculations are no longer cutting edge research topics**.

At the intersection of cutting-edge computational physics and modern plasma physics is a set of **Billion Dollar Questions**. (In general, one should not put currency values to such things).

These **Billion Dollar Questions** need huge investments in experimental and observational programs as well as the very latest in computational physics research.

Space Physics Examples: Parker Solar Probe

- Paker Solar Probe. "The primary science goals for the mission are to **trace how energy and heat move through the solar corona** and to explore **what accelerates the solar wind** as well as **solar energetic particles**."
- The Probe will collect detailed measurements of electric and magnetic fields as well as detailed distribution functions of particles.
- The solar wind plasma is nearly collisionless. It is likely that a proper understanding of kinetic physics (at the level of the Vlasov-Maxwell equations) will be needed to fully understand the physical processes.
- Cutting-edge simulations will be critical to this. Serious research into numerics of Vlasov-Maxwell needs to be done and very large simulations need to be run.

Many other missions are active and planned: BepiColombo to Mercury; Juno to Jupiter.

- Much of the deep understanding of plasma processes in solar system planets (magnetospheres, ionosphere) can only be gained from detailed modeling: global kinetic modeling is likely impossible. **How to incorporate some kinetic effects into fluid models?**

Fusion Physics Examples: Building a working thermonuclear fusion reactor.

- The Iter project aims to build the world's largest tokamak, a "magnetic bottle" to contain super-hot plasma and heat it to ignition temperatures.

There are other major fusion efforts around the world:

- Beautiful stellarators (and Wiki article) that may have better properties than tokamaks and provide a faster route to fusion energy
- High-field based compact tokamaks; field-reversed configurations; spinning magnetic mirror machines; etc

There are major unsolved problems in the basic physics of fusion machines. Most of these can only be answered by large-scale computing and much of the numerical tools have not yet been fully developed.

The Scientific Discovery through Advanced Computing program in fusion has large projects that address the very serious **Billion Dollar Question**: will controlled fusion be eventually possible?

- The numerics research here is focused on gyrokinetic and even full kinetic understanding of fundamental turbulence and transport processes in the tokamak. **These equations are very difficult to solve!**
- Disruptions are dangerous processes that can "kill" certain fusion machine: large-scale MHD simulations are needed. Significant new research is being done in new numerical methods and application of existing MHD codes to such problems.
- Runaway electrons (relativistic high-energy electron beams) can drill holes in fusion machines. See SCREAM project and special PPCF issue.
- Very serious! **Will need huge kinetic calculations**. Also, the formulation of self-consistent coupling between the runaway electrons and MHD is not complete. See review by [Boozer2015].

These are only selection of problems I am directly familiar with. I hope it gives you a flavor and understanding why computational plasma physics is such a serious and important field!

CHAPTER

FOUR

POTENTIAL SECOND-YEAR THEORY OR THESIS PROJECTS

- Expanding box simulations of solar wind-like plasma to study electron and ion temperature anisotropy instabilities and their interplay with turbulence.
- Shearing box simulations to study accretion disk instabilities and turbulence, e.g., MRI.
- Global special and/or general relativity simulations of compact object accretion disks and magnetospheres.
- High beta fluid turbulence to study the role of Alfven wave disruptions due to temperature anisotropy instabilities.
- Self-consistent test particles for studying energetic particle production in shocks, reconnection, and global simulations.
- General turbulence studies to identify ion vs electron heating, and the heating mechanism/signature across a range of plasma parameters.

CHAPTER

FIVE

SOME HISTORICAL NOTES ON HARDWARE AND KEY PAPERS

The first and perhaps greatest pioneer in computer hardware was Charles Babbage. He essentially, ab-inito, designed a series of mechanical computers, culminating in the Analytical Engine. Most of Babbage's machines were not built in his lifetime. However, his design for the Analytical Engine contains all the modern architectural details found in our processors (of course, Babbage worked with mechanical machines and not electronics). By a stroke of misfortune (probably as Babbage never published anything), Babbage's ideas were not widely known, and especially his designs fell into obscurity. They were only rediscovered in 1960s, much after the modern von Neumann architecture was designed. That two independent designs made a century apart should be so similar is remarkable.

Allan Bromley is the credited for rediscovering Babbage's legacy. See his paper in Annals of the History of Computing for a detailed overview of the Analytical Engine. Babbage's Difference engine has been built twice now. See Computer History Museum page. Babbage also designed an extraordinary printer which was also built by the Science Museum. London. See BBC news report.

The von Neumann architecture is named after the polymath John von Neumann. He worked on his designs over years, including at the Institute of Advanced Study where he designed a machined called the IAS Machine. von Neumann and Herman Goldstine wrote the first major paper on error analysis of Gaussian elimination, bringing numerical analysis into the mainstream of applied mathematics.

Moore's Law continues to hold and number of transistors on-chip are doubling every two years. However, we are hitting limit of clock speeds now and the way towards faster computing is via multi-core machines. Many companies ship chips with large number of cores. For example, Intel's Cascade Lake architecture has upto 28 cores. AMD's Epyc chips come with up to 64 cores. These are now becoming available in various supercomputing centers (including Princeton Research Computing).

CHAPTER SIX

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